S.Ü. Müh.-Mim. Fak. Derg., c.23, s.1-2, 2007
J. Fac.Eng.Arch. Selcuk Univ., v.23, n.1-2, 2007

A MATHEMATICAL PATTERN OF THE RELATION MAN-ENVIRONMENT, REPRESENTED BY ANTICIPATORY SYSTEMS

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ABSTRACT: In a pair of mixed advanced-retarded differential equations we consider man as being the master system and the environment as the slave system. This is the case when man's behaviour is dependent on a future state of the environment, while the environment's rate of change is dependent on a past state (hence a past action) of man. Man, as the master system, should take forecasts into consideration. The mathematical model used in the paper pertains to the modelling of interaction of two systems, such as the observation operator, the pragmatic operator, and especially the differential system of equations, in case the reciprocal influence between two systems manifests itself, throughout their evolution, with a time difference.

INTRODUCTION

In a general meaning, a dynamic system **S** is characterized by the following features: a) the set E of its elements; b) a set of internal relations R_i between its elements and a set of external relations R_e between the elements of the system and its environment; the inputs and the outputs of the system belong to R_e; c) all these sets E, R_i, R_e, are variable in the course of time; d) a finality is associated with the three sets. [Constantinescu, P. Otlacan, 1984]

We have to add that the subsets of the elements of E could constitute systems themselves, as subsystems of **S.** Between the subsystems of **S** and other elements or subsystems of **S** there are also internal relations. The subsystems of **S** must have the same finality as the whole system **S**.

A model of the system **S** has two components: the system state and the movement or organization law. A functional mathematical model of a system **S** links the system state at a given moment with its movement law by means of an equation. Usually, this equation is an integral or a differential equation. If the system state at a moment *t*, considered as the present moment, is a vector $x(t)$ having the components $x_i(t)$, i=1, 2, …, n, which are the parameters of the state, this state resulted from an initial state $x_0 = x(t_0)$, $t_0 < t$, and from the action of the inputs on the system in the interval of time $[t_0, t]$. The inputs are described by a vector function $g=g(g_1, g_2, \dots, g_m)$, each $g_k = g_k(\tau)$ being a real time function defined on the interval $[t_0, t]$:

$$x(t) = \mathop{F}_{\tau=t_0}^{t} [x(t_0); g(\tau)] \qquad (1.1)$$

Since before the initial state $x(t_0)$ at some prior moment $t_1<t_0$, another state $x(t_1)$ could be regarded as an initial one, the system can be considered as having an infinite memory; then the input history $g(\tau)$ is defined for $\tau \in (-\infty, t]$. In a natural manner, we believe that the system memory decreases in time, the system has a *fading memory* and, as the initial state $x(-\infty)$ is unknown, we can leave it aside.

The general correspondence from the history of inputs to the present state of the system is represented by a *constitutive functional F*:

$$x(t) = \mathop{F}_{\tau=-\infty}^{t} [g(\tau)] \qquad (1.2)$$

Hypotheses about the functional *F* and the vector function *g* led us to integral and differential expressions of the relation input-state of the system [Otlacan, 2004].

We consider that any state $x(\tau)$ with τ some moment prior to t_0 , therefore any state before the present state, has its role in the history of inputs and we can find it in $g_k(\tau)$.

Causal systems and anticipatory systems

A system whose present state is determined only by the history of its past states and past inputs is a causal system. The above formulae 1.1 and 1.2 are general representations of causal systems.

There are also other general modalities to express the constitutive equation of a causal system. In a discrete variant, at a next moment t+1, the next state $x(t+1)$ of the system is given by a recursive vector function depending on its past and present states [Dubois, 2003]:

$$x(t+1) = R(\ldots,x(t-2), x(t-1), x(t); p), \qquad (2.1)$$

where p is a set of parameters to be adjusted.

We quote the following definition from Dubois' paper: "An anticipatory system is a system for which the present behaviour is based on past and/or present events but also on future events built from these past, present and future events".

Writing $x^*(t+1)$, $x^*(t+2)$, … as predicted states for some future moments, the state $x(t+1)$ that the system will reach at the moment t+1 has the following representation:

$$x(t+1) = A(\ldots, x(t-2), x(t-1), x(t), x^*(t+1), x^*(t+2), \ldots; p) \qquad (2.2)$$

Robert Rosen's definition from his book *Anticipatory Systems* [1985] could be regarded as a classical definition: "An anticipatory system is a system containing a predictive model of itself/of its environment, which allows it to change state at an instant in accord with the model's prediction pertaining to a later instant."

Dubois himself classifies the anticipatory systems: a *weak anticipatory system* is depicted by the equation 2.2, where the future states x^* are predicted or imposed, while for a *strong anticipatory system* a formula of the following type is true:

$$x(t+1) = A(\ldots, x(t-2), x(t-1), x(t), x(t+1), x(t+2), \ldots; p), \qquad (2.3)$$

which means that the system "computes its future states from itself and not from a model-based prediction." [Dubois, 2003]

On the other hand, the anticipatory system could have a *simple incursion*, with only one state imposed at the next instant, or it could be a *hyper-incursive system*, having to choose from several future possible states.

We consider that many geological systems are strong anticipatory systems, their evolution following their own physical laws. Biological systems, whose evolution follows the genetic code, are also strong anticipatory systems.

School and the human systems of education, by their basic status, are hyper-incursive and also weak anticipative systems. The economic and social systems are also hyper-incursive, with the observation that the proposed future states are dominated by numerous interests.

A Romanian mathematician, Academician Solomon Marcus, writes in his book entitled "Timpul" (The Time), "the future now is not seen as a one and only itinerary which modelling proposes to approximate, but the future is presented as a never ending set of possible alternatives which may depend on the past, but can never be completely determined by it".

A system could make anticipations by means of predictions or prospective strategies about itself or about its environment and this kind of anticipation is named an *exo-anticipation*, in contrast with *endo-anticipation*, that regards the anticipation "embedded in the system about its own behaviour" [Dubois, 2003, p.112]. The two notions, exo and endo-anticipation, correspond to weak and strong anticipation, respectively.

Modelling interactions of system; operator of observation

We consider the case of two systems: one of them is the system **S**, the observer, and the other system **S′**, representing an object from the environment of **S** which is observed by **S**. The result of the observation at a moment t is an image $\eta(t)$, a vector in which the features of **S′** are written in a certain order. This vector-image $\eta(t)$ is created in an interval of time by the contribution of two sources: 1) the history of the evolution of the observed object, that is a pdimensional time function, $\Phi(\tau) = (\xi_1(\tau),\ldots, \xi_p(\tau))$, defined for every $\tau \leq t$; 2) the observer's capacity of reception, that has a history, too. The history of the observer's biological and intellectual characteristics and the memory of the material apparatus are expressed by a $(m\text{-}p)$-dimensional function $\Psi(\tau) = (\xi_{p+1}(\tau),\ldots, \xi_m(\tau))$, also defined for $\tau \leq t$. The future of $\eta(t)$, namely $\eta(t+\Delta t)$, is under the impetus of the more or less recent history of the couple observer-observed object.

Robert Vallée [1975] gave the following formula for the "functional paradigm" in the formative process of the image of an object, taking $\xi = (\xi_1(\tau),\ldots, \xi_p(\tau),\xi_{p+1}(\tau),\ldots, \xi_m(\tau))$:

$$\eta(t) = \int_{t_0}^{t} w(t-\tau)\xi(\tau)d\tau \qquad (3.1)$$

E. Otlacan [2000] demonstrated that this formula, where the weight-function $w(t\text{-}\tau)$ is also a vector function, only gives an approximation of the vector image $\eta(t)$. We proved a formula for the trend of the evolution of this $\eta(t)$, that is its derivative $\eta'(t)$, which is:

$$\eta'(t) = \int_{t-\lambda}^{t} K(\tau)\xi''(\tau)d\tau\,, \qquad (3.2)$$

where $\xi''(\tau)$ is the second derivative of the history $\xi(\tau)$, $\tau \in [t\text{-}\lambda, t]$ and $\lambda>0$.

Without reference to any calculus formula, Vallée [2004] deals with a general observation operator **O**, that, by acting upon the function of time ξ, results in a new formula of time, namely η:

$$\mathbf{O}(\xi) = \eta \qquad (3.3)$$

The operator **O** cannot be always inverted, but Vallée defines the reciprocally inverse operator $\mathbf{O}^{-1}$, $\mathbf{O}^{-1}(\eta)$ being the set of all ξ whose image by **O** is η.

The causal operator that acts upon the function η and gives a function of time ξ describes the evolution of the decisions which the observer takes. Vallée's *pragmatic operator* is marked by **P** and we find it in a chain of equalities:

$$\mathbf{D}(\eta) = \mathbf{D}(\mathbf{O}(\xi)) = \mathbf{DO}(\xi) = \mathbf{P}(\xi) = \zeta \qquad (3.4)$$

Hence, the pragmatic operator is the superposition of the two operators, the operator of decision over that of observation. We can see that in the formula 3.1 and in Vallée's considerations about the decision operator (superposition of the formulae 3.4, 3.3, 3.1), the observer and the observed object work as causal systems.

Taking a decision means proposing a future state $x^*(t+\Delta t)$ of the system, and for that reason we relate the concepts of observation, decision and pragmatic operator to the anticipatory systems.

We shall further present another case when two systems condition each other at certain intervals of time.

Master system and slave system – general model and applicable formulae

The evolution of an anticipatory system can be expressed by differential equations. Therefore we have two possible general forms according to the type of anticipation, weak or strong (Dubois, 2003):

$$\frac{dx(t)}{dt} = A\big[x(t-\tau), x(t), x(t+\tau); p\big] \quad (4.1)$$

$$\frac{dx(t)}{dt} = A\Big[x(t-\tau), x(t), x^*(t+\tau); p\Big] \quad (4.2)$$

Here $x^*(t+\tau)$ is taken from a model of the system, which could be achieved or not; the number $\tau>0$ is a step of time, a well-chosen "shift". We have to notice that the parameter p, the manner in which the functional A depends on this parameter have a great importance, as well as the dependence of A on the proposed future state x^*. In certain mathematical hypotheses these formulae could become more precise.

In one of our papers [Otlacan, 2005] we proved that an approximate formula could link a future state $x(t+\lambda)$, $\lambda>0$, to the present state $x(t)$ and to the history of the input that acted upon the system in an interval of time [t-λ), t+λ)], from the past t-λ to the future moment t+λ:

$$x(t+\lambda) \cong x(t) + a(t+\lambda)g(t+\lambda) + \int_{t}^{t+\lambda} q(\tau)g(\tau)d\tau + \int_{t}^{t+\lambda} d\tau \int_{t-\lambda}^{t} p(\tau,\theta)g(\theta)d\theta \quad (4.3)$$

The vector function g expresses the history of the inputs in the respective intervals of time, but the vector functions a, p, q are introduced by users of this formula, therefore by specialists. These functions may be obtained by statistical methods.

The formula 4.3 is demonstrated in conditions of differentiability of the input function and of the constitutive functional as well.

But if there are two systems which condition each other in the sense that the future of the latter is important for the present of the former and the past of the former has a decisive role in the evolution of the latter, then for these two systems Dubois associated a pair of "mixed advanced-retarded differential equations" [Dubois, 2003]:

$$\frac{dx(t)}{dt} = F[y(t+\tau)] - ax(t) \quad (4.4)$$

$$\frac{dy(t)}{dt} G[x(t-\tau)] - by(t) \quad (4.5)$$

There exists a logical reasoning in naming *master system* the system with the present state $x(t)$, as this system has the evolution of its present state built by a future state of the other system; on the other hand, the *slave system* has the evolution of its present state imposed by a past state $x(t-\tau)$ of the master system. Dubois asserts that the two systems are complementary systems and that there are circumstances in which they might synchronize.

What happens if the master system ignores the future of the slave system? From the equation 4.4, with a a constant number, by integration we obtain $x(t) = ke^{-at}$, a result that could be interpreted as being a continuous decrease of the state parameters of the master system. A similar result is deduced for the slave system if it ignores the past of the master system, but in this case it would be an independent behaviour of this system.

Man and the environment as a pair of retardation and anticipatory variables

We can think of two possible hypostases:

For the first, the master system is man having the present state $x(t)$ and the slave system is the environment, with the present state $y(t)$. The equation 4.4 says that man evolves in a certain state having in mind what the environment will be like in a future phase. He has to make plans, being aware that the evolution of the state will depend on the new future state of the environment. On the other hand, the present state of the nature / environment and its evolution depend on what man leaves behind him. We already must ask the question: does man think along these lines? Unfortunately, not always!

Man's complement, the environment, has, at least partially, a rate of change given by the equation 4.5. We consider this result in view of the fact that the present behaviour of the environment results from what man was and did some time ago. The environment has three components: natural, social and informational. For each of these components we could illustrate the situation in which man is master and the environment is slave. Science and the means of communication play an outstanding part in man's state. Examples are such natural phenomena as floods, storms, hurricanes, earthquakes, which can be forecast. They will occur at the moment $t+\tau$, but man, knowing this, acts upon his state $x(t)$ with a speed $dx(t)/dt$, according to the equation 4.4.

In general, the forecast on the environment plays a prominent part in the evolution of the state of humankind, which represents an argument for the interpretation of differential system of equations 4.4, 4.5.

Science tries to increase the interval of time from t to $t+\tau$, by augmenting the number $\tau > 0$. We read that the Japanese scientists are currently working on a supercomputer with a software able to achieve weather forecast 30 years in advance.

We have presented above arguments for the situation in which man is the master system.

In a second hypostasis, inverting the parts played by the two systems, let us consider the environment the master system, and man the slave. This means that $y(t)$ is the present state of man, $x(t)$ is that of the environment at the same moment t. We try to find these components of the environment which could evolve depending on a future state $y(t+\tau)$ of man. First we think of the informational environment. Obviously, the evolution of man expressed by the derivative $dy(t)/dt$ starts from his state $y(t)$ and from the knowledge of the past of the information technology. The interest of humanity in knowledge will be an interest in the evolution of the resources of information technology, but the latter is conditioned by man's high standard of living. Thus, not only the informational environment is involved in this situation of the environment as master system, but the social environment in its entirety. This hypostasis of the environment as master system and man as slave system might be better applied to the social component of the environment, when the evolution of the social system focuses on a future state of humankind. The future of information technology in man's environment depends on man's good economic, social, educational present. And the evolution of the social environment, such as of the informational environment, must start with a man's project.

As regards the educational system, The Romanian University of Sciences and Arts "Gheorghe Cristea" of Bucharest, aims at ensuring the quality, dynamics and efficiency of higher education, at extending the co-operation within the European and international area, at promoting the mobility of the teaching staff and of the students within this large academic area [L. Cristea, 2005]. Obviously, the natural, economic and social environment will benefit from the accomplishment of these objectives.

Nowadays we live the globalization process and this is characterized by the finest informational topology which man has ever known, that is the Topology of Communication by the Internet [Otlacan, 2003]. Locally, in each place in the world the system of knowledge tries to absorb and to expand what the people from other places have at the present moment. The behaviours of countless systems interfere and condition each other.

CONCLUSIONS

The mathematical formulae here presented found developments and more concrete aspects by using sums, products, integral or differential expressions. But our intention was to put forward some interpretations related to the starting point of the theory of anticipatory systems. We observe that results of human actions upon the environment can be seen after longer periods of time, a good example being the much debated global warming. But it is undeniable that man must always look into the future of his environment and change his own state according to the evolution of the latter. By studying a constitutive equation of a system, specialists know that sometimes the constitutive functional does not depend continuously on certain parameters p (formula 4.2) and that a small error in the appreciation of this parameter p could have catastrophic consequences for $x(t+\tau)$. So, great attention to the hypothetic state $x^*(t+\tau)$!

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