

STATISTICALLY GUIDED ARTIFICIAL BEE COLONY ALGORITHM

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ABSTRACT: Artificial Bee Colony algorithm is one of the naturally inspired meta heuristic method. As usual, in a meta heuristic method, intuitively appealing way to have better results is extending calculation time or increasing the fitness evaluation count. But the desired way is acquiring better results with less computation. So in this work a modified Artificial Bee Colony algorithm which can find better results with same computation is developed by benefiting statistical observations.

Key Words: *Swarm intelligence, Meta heuristic algorithms, Artificial bee colony algorithm.*

İstatistiksel Olarak Yönlendirilen Yapay Arı Kolonisi Algoritması

ÖZ: Yapay Arı Koloni algoritması, doğadan ilham alan meta sezgisel yöntemlerinden biridir. Meta sezgisel yöntemle, daha iyi sonuçlar elde etmek için akla ilk gelen çözüm hesaplama süresini arttırmak veya uygunluk hesaplama sayısını arttırmaktır. Ancak istenilen yol, daha az hesaplama ile daha iyi sonuçlar elde etmektir. Bu çalışmada, istatistiksel gözlemlerden yararlanarak, aynı uygunluk hesaplama sayısı ile daha iyi sonuçlar bulunabilen Yapay Arı Koloni Algoritması, geliştirilmiştir.

Anahtar Kelimeler: *Sürü zekası, Meta sezgisel algoritmalar, Yapay arı kolonisi algoritması.*

INTRODUCTION

Genetic algorithms (GA) (Holland, 1975) had marked an era in the solution of NP hard problems. It is one of mostly used population based algorithm but in recent years, swarm intelligent is also used in population based algorithms and has attracted huge attention among the researchers. Swarm intelligence algorithms are inspired from collective behavior of animal groups like ant colonies, flocks of birds or bee swarms. This special type of the population based algorithms are referred as swarm intelligence. The secret of the success of the swarm intelligence is "self organization". In a self organization, individuals in the population are specialized to fulfill a special task without under control of a centralized authority to accomplish a global task. Swarm intelligence mostly used to solve non linear functions with multi local optimum and combinational optimization tasks. Particle swarm optimization (PSO) which was introduced by (Kennedy and Eberhart, 1995) is another popular swarm intelligence based method. PSO have been inspired from collective behavior of bird or fish groups while moving together. Ant colony algorithm (ACO) (Dorigo *et al.*, 1991) which simulates the behavior of ants to find best route to carry foods from source to home is another popular swarm intelligence method. Bee colonies are also good examples for swarm intelligence. In a bee colony there are specialized bee types for specialized task. For example, employee bees fly to food sources and dances in the hive according to position and the amount of the food. Onlooker bees watch the dance of the employee bees and decide

which food resource to go so they can select the food sources which they can gather more food with less energy consumption. (Drias *et al.*, 2005) have introduced a bee colony inspired algorithm and referred as “Bees Swarm Optimization” and have tested the algorithm on MAX-W-SAT (the maximum weighted satisfiability) problem. Yang introduced another bee oriented algorithm and called as Virtual Bee Algorithm and tested it with two dimensional problems under one agent and multi-agent conditions (Yang, 2005). Teodorovic proposed bee swarm intelligence based algorithm and tested to solve complex traffic and transportation problems (Teodorovic', 2003; Lucic and Teodorovic' 2002).

The main motivation of this work is Artificial bee colony (ABC) algorithm which simulates the foraging behavior and collective work of different kinds of bees by (Karaboğa, 2005). The algorithm tested on multimodal and multi-dimensional numerical optimization problems. ABC was firstly developed to solve numerical optimization problems and the performance compared to GA and particle swarm inspired evolutionary algorithm (PS-EA) (Basturk and Karaboga, 2006; Karaboga and Basturk, 2007). Performance of differential evaluation (DE), PSO and evolutionary algorithm (EA) on basic numerical functions are also tested against ABC (Karaboga and Akay, 2008; Karaboga and Basturk, 2008). ABC algorithm is also used to train artificial neural network weights (Karaboga and Akay, 2007; Karaboga *et al.*, 2007), classify medical patterns, clustering (Karaboga *et al.*, 2008; Ozturk and Karaboga, 2008) and solving travelling salesman problem (Shrivastava *et al.*, 2015).

In this work our goal was improving the performance of the ABC algorithm without increasing the maximum fitness evaluation count. Performance of the proposed method is investigated for real-parameter optimization on both basic and composite functions presented at the Congress of Evolutionary Computation 2005 (CEC05). In Section 2 ABC algorithm was introduced. In Section 3 proposed modifications on ABC algorithm are introduced. In Section 4 test results for different dimensions of CEC05 problems are presented and in Section 5 test results are discussed.

ARTIFICIAL BEE COLONY ALGORITHM

Metaheuristic algorithms are developed to solve combinational optimization problems like travelling salesman problem or vehicle routing problem but today they are also used to solve real parameter estimation problem which can be described as finding best parameter values of a function which minimizes or maximizes the function. For example in equation 1 if it is wanted to find the best x and y values which minimizes the function under circumstances of $-3 < x < 5.5$ and $y > 12$ then this problem can be described as a real parameter optimization problem.

$$f(x, y) = x^2 - 2^y + 2xy \quad (1)$$

In this work the proposed method tested on real parameter optimization problem. ABC algorithm searches the global search space to find suitable parameter values by three types of bees (or agents) which are listed below.

Employee Bee: A food source (or possible solution) is assigned to an employee bee. The mission of an employee bee is giving information about the particular food source which it is assigned to onlooker bees. This information is the food amount (quality of the solution). After a food source has not enough resources anymore then the employee bee which assigned to that source become scout bees.

Onlooker Bee: This type of bees search better food sources around the employee bees. The idea behind is better solutions should be around the best solutions. At this point employee bees guide to onlooker bees to better solutions.

Scout Bee: Scout bees are assigned to find new food sources that are not found by employee bee. So they fly to far away that are not visited yet. After they found a new source they become employee bees.

The flowchart of the ABC algorithm is illustrated in Figure 1.

STATISTICALLY GUIDED ARTIFICIAL BEE COLONY ALGORITHM

A weak point of the ABC algorithm is that it searches a better solution near the current solution by modifying only one parameter at a time. But some of the other metaheuristic algorithms changes more than one parameter to speed up the convergence. However it is tried to modify more than one parameters in an ABC variant (Akay and Karaboga, 2010) but the algorithm uses another parameter which decides to make modification more than one parameter. The idea behind the proposed method finding value which is very close the optimal value to use on all agents as the second parameter modification. The problem is calculating that kind of near optimal value? At this point statistical methods come to help. Proposed method detects the near optimal values in three steps:

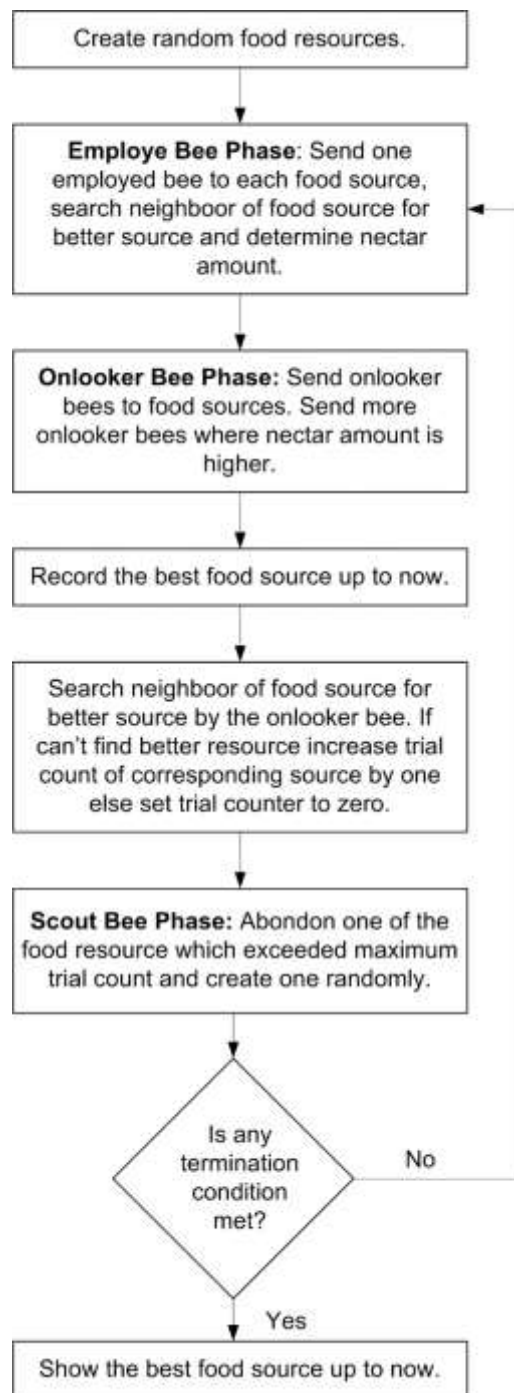


Figure 1. Flowchart of the ABC algorithm.

First step is detecting first 10 best fitness valued solutions. We use 100 as population count and 50 for food sources count. Solutions (food sources) are sorted by their fitness values and parameter values of first 10 solutions are reserved for further steps.

At the second step standard deviation (SD) values of the each parameter are calculated from the best fitness valued solutions that are detected and reserved in step 1. SD is a measurement technique to understand how much an array of variable are different from each other. SD is calculated in 3 steps. In first step average value of the array is calculated by equation 2. In second step variance of the array is calculated by equation 3 and in last step SD is calculated as square root of the variance in equation 4.

Flowchart of the proposed algorithm is presented in figure 2 and pseudo code is presented in figure 4. Changed parts are written in red.

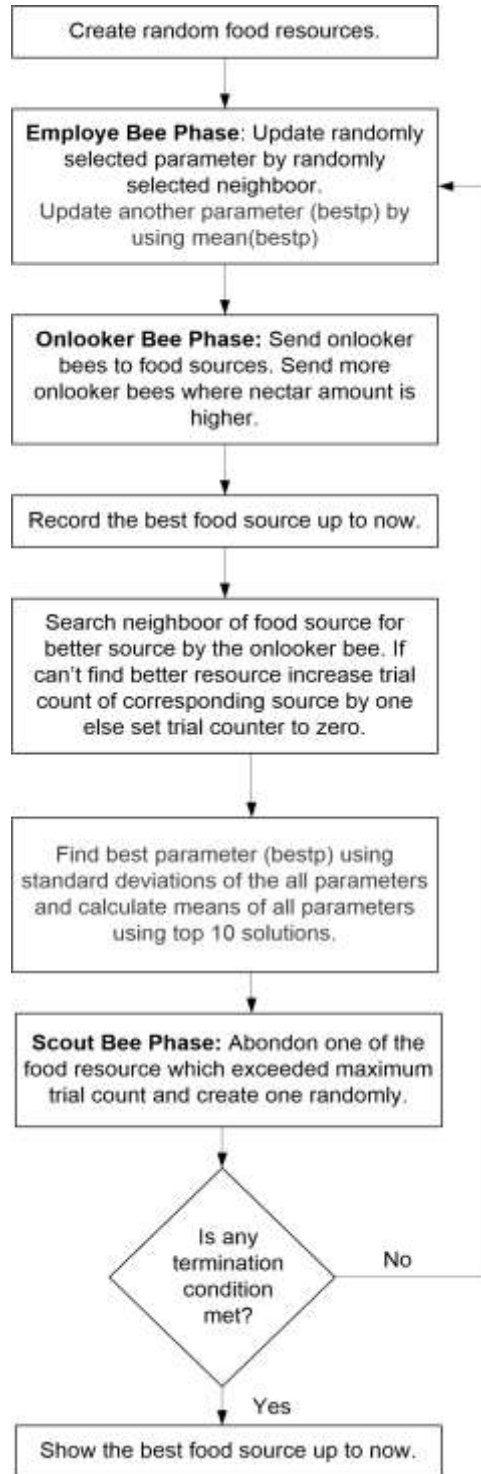


Figure 2. Flowchart of proposed ABC algorithm

$$M = \frac{1}{n} \sum_{i=1}^n a_i$$

(2)

$$VAR = \frac{1}{N} \sum_{i=1}^N (a_i - M)^2 \tag{3}$$

$$\sigma = \sqrt{VAR} \tag{4}$$

SD values are calculated for each dimension of the food sources to find the parameter which is closer to optimum value. But how can it be decided that mean value of a parameter is near the optimum value only by looking the SD value of the parameter? Solution of that problem can be shown by an example. The box-plot representation in figure 3 is acquired by the parameter values of first 10 best solutions in 11.cycle of ABC for the 10 dimensions sphere function which is described by Equation 5.

$$f(x) = \sum_{i=1}^D x_i^2 \tag{5}$$

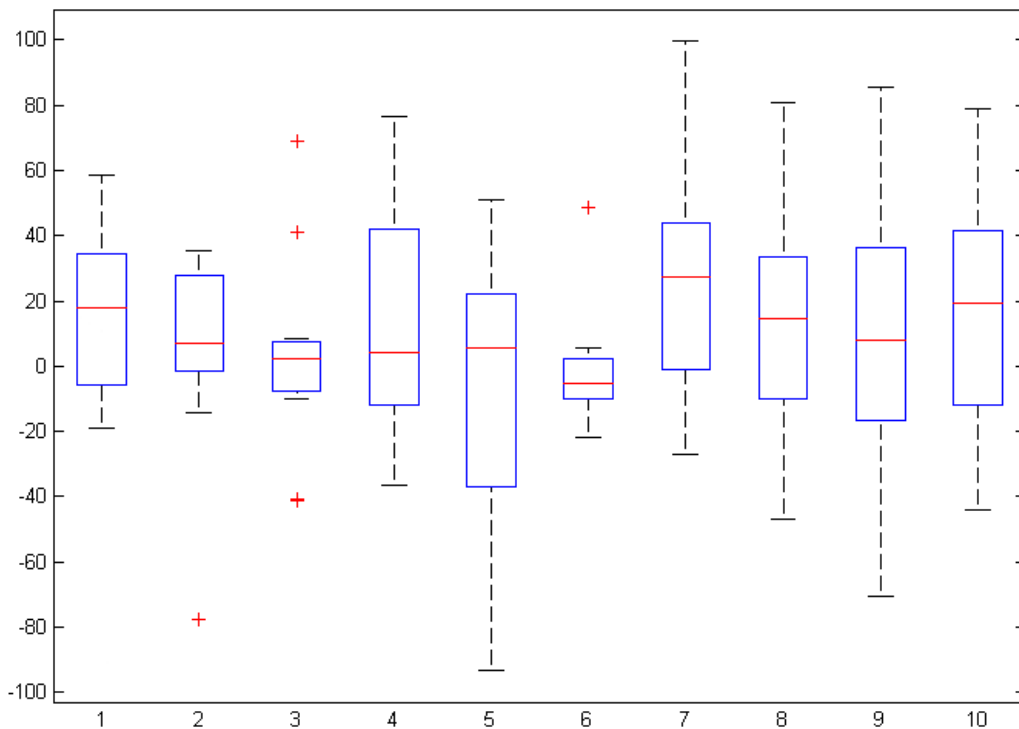


Figure 3. Box-plot representation of first 10 best solutions of the sphere function in 11.cycle.

Each box represents a parameter (or dimension). Short box represent small SD values and tall box represents big SD values. Values in figure 2 are summarized in Table 1. Columns in Table 2 represent the parameters and first two rows represent SD values, mean values of the first 10 best solutions respectively. The row referred as "Position on SD" represents the parameter's position when the parameters sorted by their SD values in ascending order and the row referred as "Position on Fitness" represents the parameter positions when the parameters are sorted by their distance of means values to optimum values. For example parameter 6 (D6) has a SD value 18 and when the parameters are sorted by the SD it's position is 1 because it has the lowest SD value among the parameters. The mean value of the D6 -1.82 and its optimum value for sphere function in equation 5 is zero. So the distance to optimum

value is -1.82 and its "Position on Fitness" value is also 1. The same relation can be seen other dimensions. So at this point it can be said that "Position on SD" values are related to "Position on Fitness" values. More clearly it can be said that mean value of the minimum SD valued parameter is very close to its optimum value. This assumption is the key concept of the proposed method. So we detect the best parameter in every cycle by SD values and use its value for second parameter modification.

Table 1. Summarized values for figure 3.

Description	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
Standard Deviation	26.9	30.69	30.14	36.52	44.42	18	36.7	35.84	40.02	39.55
Mean	16.78	5.77	2.85	11.69	-8.86	-1.82	23.79	12.13	7.94	15.48
Position on SD	2	4	3	6	10	1	7	5	9	8
Position on Fitness	9	3	2	6	5	1	10	7	4	8

Modified parts of the algorithm are shown bold. In modified part *"/*Finding appropriate parameter*/"* best fitness valued solutions are sorted and first 10 of them are used to calculate SD values of the parameters. Minimum SD valued parameter number is assigned to "bestp" variable. The other modified part of the algorithm is *"/* Second parameter modification*/"* where the bestpth parameter of each employee bee is modified by the mean value of the bestpth parameter. Second parameter modification is similar to standard parameter modification in ABC except instead of finding a random parameter of a random neighbor it uses mean value of the bestpth parameter. Second parameter modification is used only in in employee bee phase because using it onlooker bee phase causes solutions to bias thought to same value. Second parameter modification is applied after 10 iterations from beginning in order to allow parameters to decide a near optimal value.

EXPERIMENTAL RESULTS

The proposed method tested against to standard ABC algorithm with both basic and composite functions presented at Congress of Evolutionary Computation 2005 (CEC05) in four categories. First category is fitness evaluation tests. In the test proposed fitness values of the proposed method are compared to original ABC under the same conditions. In second test methods are compared by the stability. For this purpose standard deviation of fitness values that are obtained after 30 independent runs are compared. In third test convergence speeds are tested for both methods. Last test was time consumption test. In this test methods are compared by their time consumptions.

We have used 10, 30 and 50 for problem dimensions. For a fair comparison we have used the same parameters for both methods. Population size is set to 100 and food sources size is set to 50. MaxCycle is set according to dimension size. We established 10^4 fitness evaluations for each dimension by calculating MaxCycle value in Equation 6.

$$MaxCycle = \frac{D \times 10000}{pop_size} \quad (6)$$

where "D" represents the dimensionality and "pop_size" represents population number of the ABC algorithm.

<pre> Create random food sources x[i,j] where i represents the solution number and j represents the parameter evaluate the fitness of the all solutions for cycle=1 to MaxCycle do begin /*Employe bee phase*/ for fnum=1 to maxfoodnum do begin Select random neighbour a, random parameter b and generate random number c in interval[-1,1] t = x[fnum,b] + c(x[a,b]-x[fnum,b]) if (t>max[b]) t= max[b] if (t<min[b]) t= min[b] x[fnum,b] = t /* Second parameter modification*/ if cycle>10 then begin x[fnum,bestp]=x[fnum,bestp]+c(x[fnum,bestp]+mea n[bestp]) /*"bestp" is calculated below in /*Finding appropriate parameter*/ phase end calculate the fitness of x[fnum] if fitness is better than old one then trial[fnum]=0 else begin revert x[fnum,b] to old value trial[fnum]++ end end Calculate the probablity values of each food source p /*Onlooker bee phase*/ fnum=1 tnum=0 while fnum<=maxfoodnum begin generate random number r in interval [0,1] tnum++ if r<p[tnum] then begin </pre>	<pre> Select random neighbour a, random parameter b and generate random number c in interval[-1,1] t = x[tnum,b] + c(x[a,b]-x[tnum,b]) if (t>maxfnum) t=maxfnum if (t<minfnum) t=minfnum x[tnum,b] = t calculate the fitness of x[tnum] if fitness is better than old one then trial[tnum]=0 else begin revert x[fnum,b] to old value trial[tnum]++ end fnum++; end if tnum>maxfoodnum then tnum=0 end /*Scout bee phase*/ find maximum valued indices z in trial if trial[z]>maxtrial then generate random food source instead of x[z] /*Finding appropriate parameter*/ copy all foodsources x to y //in order not to break the orignality of ABC algorithm sort y according to fitness values for i=1 to D do begin mean[i]=0; for j=1 to 10 do begin mean[i] = mean[i] + y[j,i] end mean[i] = mean[i] / 10 stddev[i] = 0; for j=1 to 10 do begin stddev[i] = (mean[i] - y[j,i])² end stddev[i] = sqrt(stddev[i]/10) end find minimum valued indices and assign it bestp end </pre>
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Figure 4. Proposed modified ABC algorithm.

Fitness Evaluation Test:

Fitness evaluation test results are obtained by the results of 30 independent runs. In "Function Number" column, corresponding function numbers in CEC05 are presented, in "ABC" column mean fitness value of the 30 independent runs are presented. Similarly in "Modified ABC" column mean fitness value of 30 independent runs are presented for proposed modified ABC algorithm. In "Better Than Classical ABC" column represents the comparison result of the mean of original ABC and Modified ABC test results. If Modified ABC is better than the original one then the column value is set to "Yes" otherwise column value is set to "No". "Statistically Significant" column represents paired t test results of the 30 independent runs at the 5% significance level. If difference between the results is statistically significance then column value is set to "Yes" otherwise the column value is set to "No".

Table 2. Test Results for 10 dimensions problems.

Function Number	ABC	Modified ABC	Better Than Classical ABC?	Statistically Significant?
1	9.47072E-17	9.94065E-17	No	No
2	18.94657779	1.662162747	Yes	Yes
3	575517.5816	763078.2204	No	No
4	1064.454258	290.0148787	Yes	Yes
5	99.87026555	12.7641938	Yes	Yes
6	1.019229712	3.042330888	No	No
7	0.134277423	0.108344259	Yes	No
8	20.30742474	20.30065673	Yes	No
9	0	0	No	No
10	28.38667902	18.26045643	Yes	Yes
11	5.623053561	5.128848372	Yes	Yes
12	44.31023529	62.94259948	No	No
13	0.035337889	0.041974926	No	No
14	3.339751658	3.350167432	No	No
15	3.15139E-06	3.75222E-05	No	No
16	151.1603956	139.3305692	Yes	Yes
17	164.9748003	155.2562062	Yes	Yes
18	543.8301847	545.9794051	No	No
19	608.1872842	550.5049652	Yes	No
20	552.1229053	550.2109244	Yes	No
21	389.3503569	367.4435637	Yes	No
22	754.4939213	737.8748394	Yes	No
23	520.2365368	528.4251605	No	No
24	200.0000027	200.0000404	No	No
25	200.0000078	200.0000572	No	No

Table 3. Test Results for 30 dimensions problems.

Function Number	ABC	Modified ABC	Better Than Classical ABC?	Statistically Significant?
1	5.10142E-16	5.30854E-16	No	No
2	6174.301225	4251.858141	Yes	Yes
3	8472701.954	8169506.466	Yes	No
4	33289.28758	30499.99252	Yes	No
5	11238.9255	9782.00526	Yes	Yes
6	3.319829264	5.95898134	No	No
7	0.061309309	0.044651046	Yes	Yes
8	20.77726229	20.79922538	No	No
9	0	0	No	No
10	331.4970905	253.7468551	Yes	Yes
11	28.31282623	27.24817996	Yes	Yes
12	8204.399412	9716.293662	No	No
13	0.316065204	0.347643753	No	No
14	12.92264784	12.87690019	Yes	No
15	5.61049E-05	0.015892386	No	No
16	292.7251296	285.3769222	Yes	No
17	359.291756	349.5481241	Yes	No
18	918.4480828	899.8375004	Yes	No
19	920.3222711	910.9465426	Yes	Yes
20	915.2599772	915.6701221	No	No
21	497.0657001	491.2270562	Yes	No
22	1075.049454	1031.820385	Yes	Yes
23	531.8314824	531.2477391	Yes	No
24	344.7514483	234.457366	Yes	No
25	395.9891783	250.784032	Yes	No

Table 4. Test Results for 50 dimensions problems.

Function Number	ABC	Modified ABC	Better Than Classical ABC?	Statistically Significant?
1	9.20521E-16	1.05959E-15	No	Yes
2	24310.18996	21763.67147	Yes	Yes
3	18745986.84	18108940.93	Yes	No
4	97433.28895	94860.89296	Yes	No
5	25355.42562	24600.06374	Yes	No
6	2.431763026	8.933342184	No	Yes
7	0.034510315	0.031968983	Yes	No
8	20.91500183	20.96498148	No	Yes
9	0	0	No	No
10	1013.368826	828.2220972	Yes	Yes
11	56.76164227	55.0178908	Yes	Yes
12	45751.19723	48587.14031	No	No
13	0.687145481	0.85773778	No	Yes
14	22.6787752	22.53280833	Yes	No
15	0.004183417	7.573192646	No	No
16	396.6940249	394.8546597	Yes	No
17	471.7846714	474.7504174	No	No
18	975.7403714	964.1504486	Yes	No
19	978.9170299	961.2534525	Yes	Yes
20	982.7706413	960.6250472	Yes	Yes
21	500.0001331	500.0000111	Yes	No
22	1159.142349	1136.929475	Yes	Yes
23	539.1241006	539.1237518	Yes	No
24	1333.38159	1308.975774	Yes	Yes
25	1332.985896	1314.894348	Yes	Yes

The fitness evaluation test results are a bit complicated so we summarized all the test tables above in Table 5.

Table 5. Summarized fitness evaluation test results.

Dimensions	Proposed method is better than the Classical ABC	Statistically Significant?	Proposed method is worse than the Classical ABC	Statistically Significant?
10	13	7	12	0
30	17	7	8	0
50	17	8	8	4

Fitness evaluation test results are encouraging. In most of the tests, proposed method is better than ABC in most cases and also when it is worse than the original ABC, the results are not statistically significance.

Standard Deviation Test

SD test are done in order to measure the stability of the methods. If two different methods compared with the same test functions by 30 independent runs it can be said that the smaller SD valued function is more stable than the other one. SD test results are presented in Table 6.

Table 6. SD test results

Function Number	D=10			D=30			D=50		
	ABC	Modified ABC	Better Than Classical ABC?	ABC	Modified ABC	Better Than Classical ABC?	ABC	Modified ABC	Better Than Classical ABC?
1	1.18E-17	1.25E-17	No	5.55E-17	6.09E-17	No	1.32E-16	1.09E-16	Yes
2	14.83463	1.238683	Yes	1856.119	1387.997	Yes	3997.128	2726.737	Yes
3	297920.5	514213.6	No	2764282	2388461	Yes	4605850	5586789	No
4	648.3193	173.9363	Yes	5957.668	5570.806	Yes	12543.29	9729.833	Yes
5	121.8133	53.2	Yes	1527.727	1456.137	Yes	1948.405	2047.314	No
6	1.955067	7.789525	No	4.327962	9.071855	No	3.73836	13.4603	No
7	0.050566	0.051547	No	0.018554	0.017983	Yes	0.007188	0.014279	No
8	0.070881	0.080921	No	0.079209	0.082064	No	0.048878	0.039768	Yes
9	0	0	No	0	0	No	0	0	No
10	5.400138	5.343894	Yes	49.31465	42.80711	Yes	98.77225	108.6589	No
11	0.744279	0.800136	No	1.680044	1.458333	Yes	2.737719	2.497108	Yes
12	37.10748	71.9975	No	4491.794	4741.499	No	15669.93	18980.61	No
13	0.016922	0.020334	No	0.103545	0.109501	No	0.141407	0.274759	No
14	0.225732	0.226375	No	0.282256	0.251902	Yes	0.216866	0.350755	No
15	1.02E-05	0.000141	No	0.000188	0.053657	No	0.022268	35.91271	No
16	15.14174	14.37557	Yes	35.33667	40.476	No	7.913829	13.1236	No
17	18.33895	17.91638	Yes	45.07728	49.80905	No	10.30309	10.30978	No
18	101.8693	95.0339	Yes	4.319544	92.88999	No	30.22302	22.35548	Yes
19	144.8556	114.7886	Yes	4.497828	21.23954	No	20.42147	21.63619	No
20	99.95363	105.9822	No	21.73962	2.925873	Yes	28.11129	19.13792	Yes
21	77.05258	99.30221	No	15.80195	26.32415	No	0.000654	1.43E-05	Yes
22	120.5941	144.6947	No	29.30139	40.16287	No	32.73313	39.21758	No
23	52.49604	46.2016	Yes	5.988237	6.586686	No	0.000999	0.000916	Yes
24	1.38E-05	0.000215	No	325.6949	183.2311	Yes	28.38805	26.49105	Yes
25	3.5E-05	0.000308	No	362.1658	201.2555	Yes	28.57924	25.10953	Yes

The SD test results are a bit complicated so we summarized all the test tables above in Table 7.

Table 7. Stability test results.

Dimensions	Proposed method is more stable than the Classical ABC	Proposed method is less stable than the Classical ABC
10	9	16
30	11	14
50	11	14

In stability test it can't be said that original ABC is more stable than the proposed method for all situation. Especially test results are close for the problems which have 30 and 50 dimensions.

Convergence Speed test:

Convergence speed are illustrated in figure 5-10 in order to present which method can find better results with less fitness evaluations.

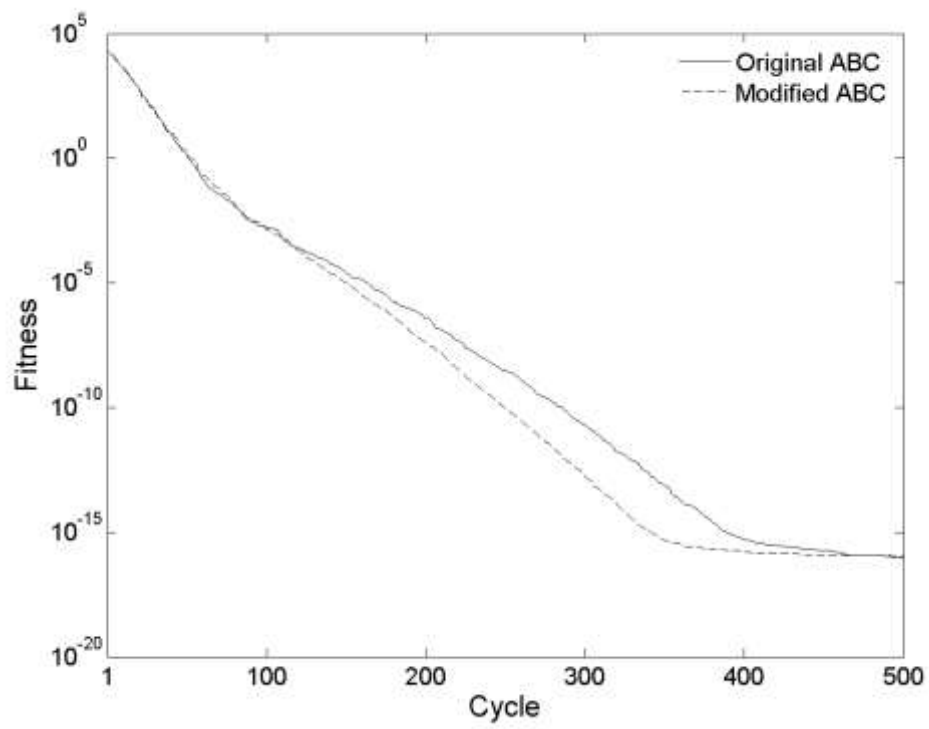


Figure 5. Convergence graphics for function 1 and dimension 10

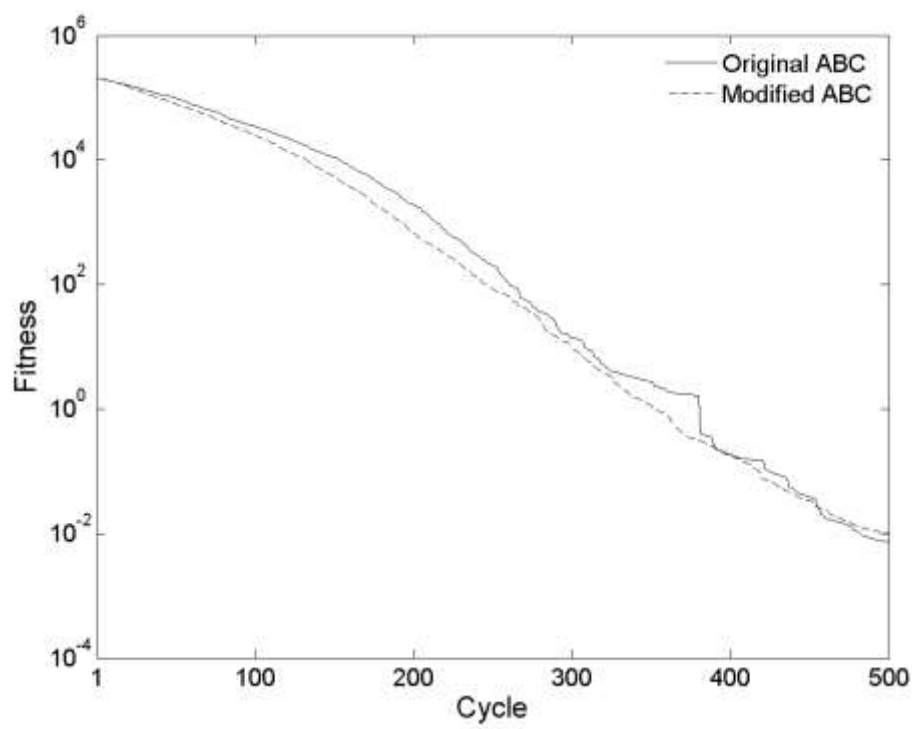


Figure 6. Convergence graphics for function 1 and dimension 50

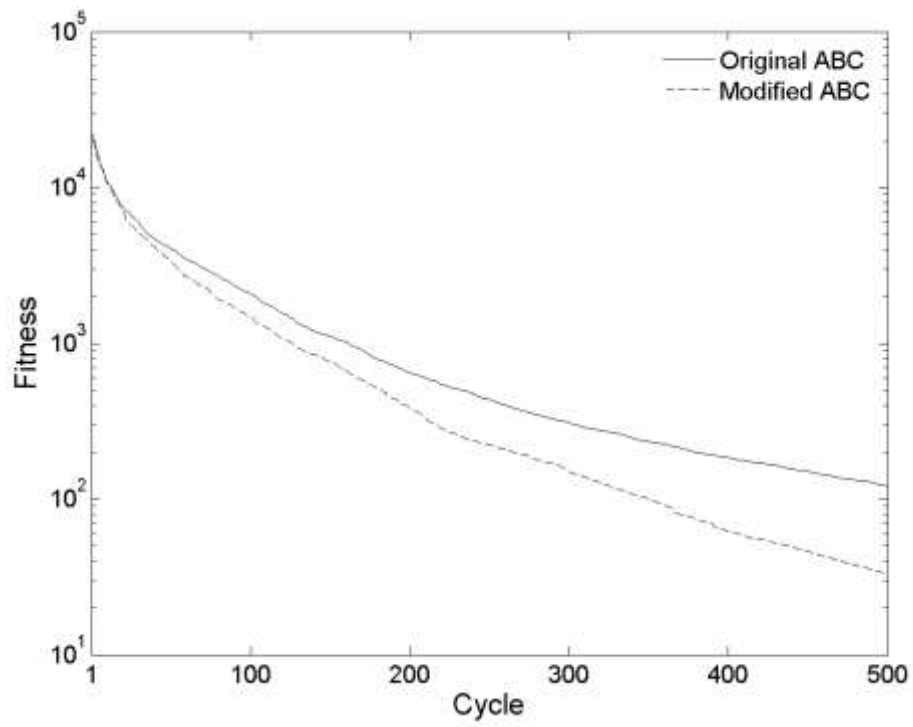


Figure 7. Convergence graphics for function 2 and dimension 10

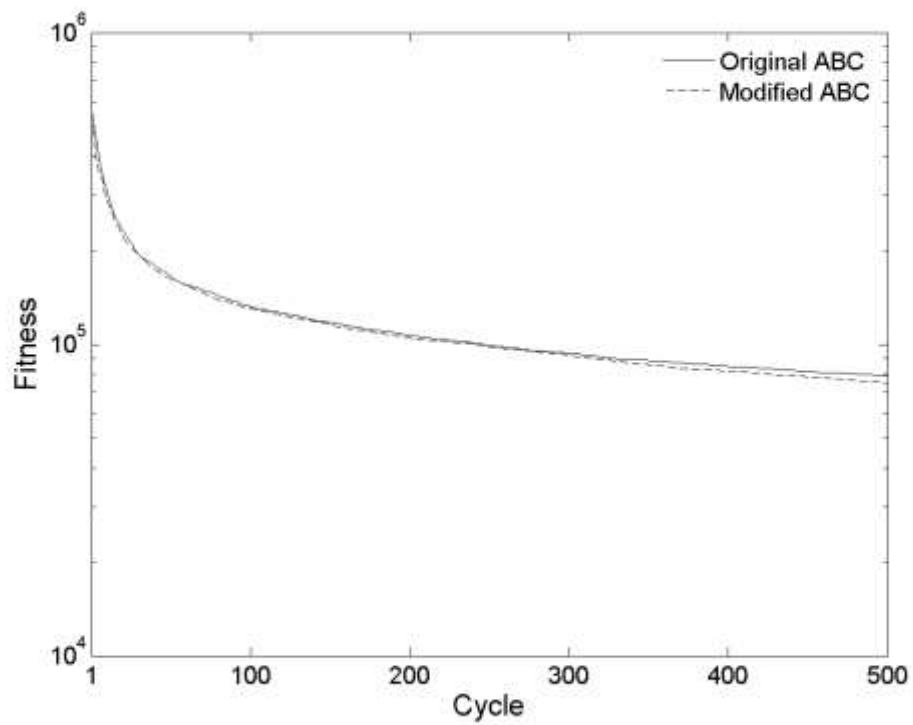


Figure 8. Convergence graphics for function 2 and dimension 50

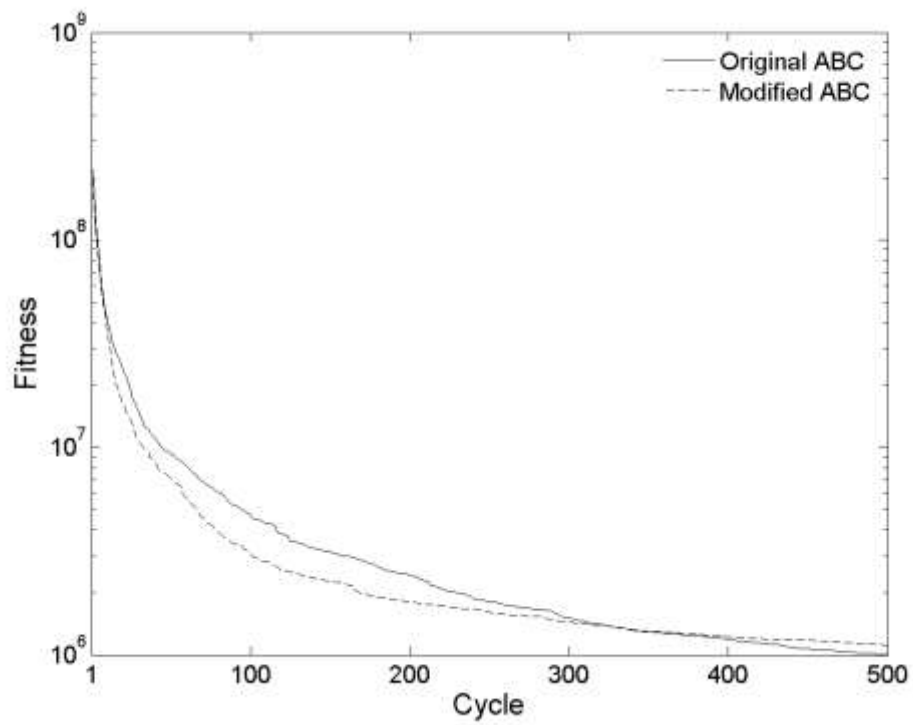


Figure 9. Convergence graphics for function 3 and dimension 10

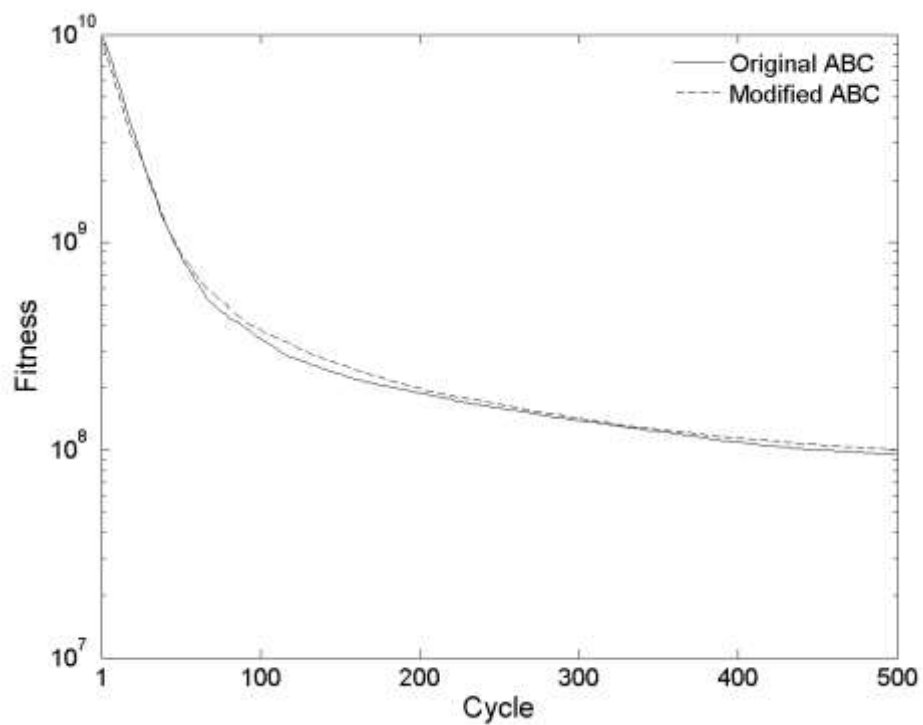


Figure 10. Convergence graphics for function 3 and dimension 50

In convergence test it can be said that proposed method find better solutions with less fitness evaluations in lower dimension sizes. When dimension sizes get larger the advantage of the proposed method is lost.

Time Consuming Test:

Developing a better method over the original one always has some additional cost. These costs may be additional fitness evaluations, additional computations or additional memory consumption. However the important rule in swarm intelligence methods is finding better fitness values with less or the same fitness evaluation counts. So proposed method uses the same fitness evaluation count and needs a little additional computation to find better fitness values. The additional computation cost is presented by a time consuming test for “Rosenbrock” function by the different dimension sizes in Table 8.

Table 8. Time consuming test results for Rosenbrock function.

Dimensions	Original ABC	Proposed Method	Increment (%)
10	9.53166	9.56286	0.327330182
30	29.7494	29.8586	0.367066227
50	51.1215	51.2931	0.335670902

When Table 8 evaluated, it can be said that proposed method needs less than 1% additional computation according to original one and this is not a significance difference.

CONCLUSIONS

According to experimental results it can be claimed that proposed method has found better result with same fitness evaluations. The proposed method needs only %0.5 additional computation and this cost can be tolerated. Some of the other modified methods also can find better results than original ABC algorithm but they need additional parameters and the values of the parameters should be set properly by the user, otherwise method may find worse results. In this perspective proposed method is a parameter-less method and can work without user interactions. Stability and fitness performance of the method can be improved method by additional modifications.

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